Homework 3 Solutions

$S_{ut} = 120$ kpsi, $\sigma_{\text{rev}} = 70$ kpsi

Fig. 6-18: $f = 0.82$

Eq. (6-8): $S'_e = S_e = 0.5(120) = 60$ kpsi

Eq. (6-14): $a = \dfrac{(f\,S_{ut})^2}{S_e} = \dfrac{\left[0.82(120)\right]^2}{60} = 161.4$ kpsi

Eq. (6-15): $b = -\dfrac{1}{3}\log\left(\dfrac{f\,S_{ut}}{S_e}\right) = -\dfrac{1}{3}\log\left(\dfrac{0.82(120)}{60}\right) = -0.0716$

Eq. (6-16): $N = \left(\dfrac{\sigma_{\text{rev}}}{a}\right)^{1/b} = \left(\dfrac{70}{161.4}\right)^{\frac{1}{-0.0716}} = 116\,700$ cycles *Ans.*

6-11 For AISI 4340 as-forged steel,

Eq. (6-8): $S_e = 100$ kpsi

Table 6-2: $a = 39.9$, $b = -0.995$

Eq. (6-19): $k_a = 39.9(260)^{-0.995} = 0.158$

Eq. (6-20): $k_b = \left(\dfrac{0.75}{0.30}\right)^{-0.107} = 0.907$

Each of the other modifying factors is unity.

$S_e = 0.158(0.907)(100) = 14.3$ kpsi *Ans.*

For AISI 1040:

$S'_e = 0.5(113) = 56.5$ kpsi

$k_a = 39.9(113)^{-0.995} = 0.362$

$k_b = 0.907$ (same as 4340)

Each of the other modifying factors is unity

$S_e = 0.362(0.907)(56.5) = 18.6$ kpsi *Ans.*

Not only is AISI 1040 steel a contender, it has a superior endurance strength. *Ans.*

6-12 $D = 1$ in, $d = 0.8$ in, $T = 1800$ lbf·in, $f = 0.9$, and from Table A-20 for AISI 1020 CD, $S_{ut} = 68$ kpsi, and $S_y = 57$ kpsi.

(a) Fig. A-15-15: $\dfrac{r}{d} = \dfrac{0.1}{0.8} = 0.125$, $\dfrac{D}{d} = \dfrac{1}{0.8} = 1.25$, $K_{ts} = 1.40$

Get the notch sensitivity either from Fig. 6-21, or from the curve-fit Eqs. (6-34) and (6-35b). Using the equations,

$$\sqrt{a} = 0.190 - 2.51\left(10^{-3}\right)(68) + 1.35\left(10^{-5}\right)(68)^2 - 2.67\left(10^{-8}\right)\left(68^3\right) = 0.07335$$

$$q_s = \frac{1}{1 + \dfrac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1 + \dfrac{0.07335}{\sqrt{0.1}}} = 0.812$$

Eq. (6-32): $K_{fs} = 1 + q_s\,(K_{ts} - 1) = 1 + 0.812(1.40 - 1) = 1.32$

For a purely reversing torque of $T = 1800$ lbf·in,

$$\tau_a = K_{fs}\frac{Tr}{J} = \frac{K_{fs}16T}{\pi d^3} = \frac{1.32(16)(1800)}{\pi(0.8)^3} = 23\,635 \text{ psi} = 23.6 \text{ kpsi}$$

Eq. (6-8): $S'_e = 0.5(68) = 34$ kpsi

Eq. (6-19): $k_a = 2.70(68)^{-0.265} = 0.883$

Eq. (6-20): $k_b = 0.879(0.8)^{-0.107} = 0.900$

Eq. (6-26): $k_c = 0.59$

Eq. (6-18) (labeling for shear): $S_{se} = 0.883(0.900)(0.59)(34) = 15.9$ kpsi

For purely reversing torsion, use Eq. (6-54) for the ultimate strength in shear.

Eq. (6-54): $S_{su} = 0.67\, S_{ut} = 0.67(68) = 45.6$ kpsi

Adjusting the fatigue strength equations for shear,

Eq. (6-14): $$a = \frac{\left(f\, S_{su}\right)^2}{S_{se}} = \frac{\left[0.9(45.6)\right]^2}{15.9} = 105.9 \text{ kpsi}$$

Eq. (6-15): $$b = -\frac{1}{3}\log\left(\frac{f\,S_{su}}{S_{se}}\right) = -\frac{1}{3}\log\left(\frac{0.9(45.6)}{15.9}\right) = -0.137\,27$$

Eq. (6-15): $$N = \left(\frac{\tau_a}{a}\right)^{\frac{1}{b}} = \left(\frac{23.6}{105.9}\right)^{\frac{1}{-0.13727}} = 56188\ cycles \quad Ans.$$

(b) For an operating temperature of 750°F, the temperature modification factor, from Table 6-4 is $k_d = 0.90$.

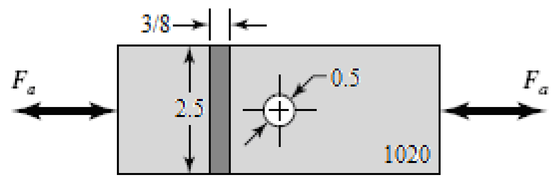
$$S_{se} = 0.883(0.900)(0.59)(0.9)(34) = 14.3 \text{ kpsi}$$

$$a=\frac{\left(f\,S_{su}\right)^2}{S_{se}}=\frac{\left[0.9(45.6)\right]^2}{14.3}=117.8\ \text{kpsi}$$

$$b=-\frac{1}{3}\log\left(\frac{f\,S_{su}}{S_{se}}\right)=-\frac{1}{3}\log\left(\frac{0.9(45.6)}{14.3}\right)=-0.152\ 62$$

$$N=\left(\frac{\tau_a}{a}\right)^{\frac{1}{b}}=\left(\frac{23.6}{117.8}\right)^{\frac{1}{-0.15262}}=37582\ cycles$$

6-14 Given: w =2.5 in, t = 3/8 in, d = 0.5 in, n_d = 2. From Table A-20, for AISI 1020 CD, S_{ut} = 68 kpsi and S_y = 57 kpsi.



Eq. (6-8): $S_e'=0.5(68)=34$ kpsi

Table 6-2: $k_a=2.70(68)^{-0.265}=0.88$

Eq. (6-21): k_b = 1 (axial loading)

Eq. (6-26): k_c = 0.85

Eq. (6-18): S_e = 0.88(1)(0.85)(34) = 25.4 kpsi

Table A-15-1: $d/w=0.5/2.5=0.2,\ K_t=2.5$

Get the notch sensitivity either from Fig. 6-20, or from the curve-fit Eqs. (6-34) and (6-35a). The relatively large radius is off the graph of Fig. 6-20, so we will assume the curves continue according to the same trend and use the equations to estimate the notch sensitivity.

$$\sqrt{a}=0.246-3.08\left(10^{-3}\right)(68)+1.51\left(10^{-5}\right)(68)^2-2.67\left(10^{-8}\right)\left(68^3\right)=0.09799$$

$$q=\frac{1}{1+\frac{\sqrt{a}}{\sqrt{r}}}=\frac{1}{1+\frac{0.09799}{\sqrt{0.25}}}=0.836$$

Eq. (6-32): $K_f=1+q(K_t-1)=1+0.836(2.5-1)=2.25$

$$\sigma_a=K_f\frac{F_a}{A}=\frac{2.25F_a}{(3/8)(2.5-0.5)}=3F_a$$

Since a finite life was not mentioned, we'll assume infinite life is desired, so the completely reversed stress must stay below the endurance limit.

$$n_f=\frac{S_e}{\sigma_a}=\frac{25.4}{3F_a}=2$$

$F_a=4.23$ kips *Ans.*

6-15 Given: $D=2$ in, $d=1.8$ in, $r=0.1$ in, $M_{\max}=25\,000$ lbf $\cdot$in, $M_{\min}=0$.

From Table A-20, for AISI 1095 HR, S_{ut} = 120 kpsi and S_y = 66 kpsi.

Eq. (6-8): $S'_e = 0.5S_{ut} = 0.5(120) = 60$ kpsi

Eq. (6-19): $k_a = aS_{ut}^b = 2.70(120)^{-0.265} = 0.76$

Eq. (6-24): $d_e = 0.370d = 0.370(1.8) = 0.666$ in

Eq. (6-20): $k_b = 0.879d_e^{-0.107} = 0.879(0.666)^{-0.107} = 0.92$

Eq. (6-26): $k_c = 1$

Eq. (6-18): $S_e = k_a k_b k_c S'_e = (0.76)(0.92)(1)(60) = 42.0$ kpsi

Fig. A-15-14: $D/d = 2/1.8 = 1.11, \quad r/d = 0.1/1.8 = 0.056 \quad \Rightarrow K_t = 2.1$

Get the notch sensitivity either from Fig. 6-20, or from the curve-fit Eqs. (6-34) and (6-35a). Using the equations,

$$\sqrt{a} = 0.246 - 3.08\left(10^{-3}\right)(120) + 1.51\left(10^{-5}\right)(120)^2 - 2.67\left(10^{-8}\right)\left(120^3\right) = 0.04770$$

$$q = \frac{1}{1 + \dfrac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1 + \dfrac{0.04770}{\sqrt{0.1}}} = 0.87$$

Eq. (6-32): $K_f = 1 + q(K_t - 1) = 1 + 0.87(2.1 - 1) = 1.96$

$$I = (\pi/64)d^4 = (\pi/64)(1.8)^4 = 0.5153 \text{ in}^4$$

$$\sigma_{\max} = \frac{Mc}{I} = \frac{25\,000(1.8/2)}{0.5153} = 43\,664 \text{ psi} = 43.7 \text{ kpsi}$$

$$\sigma_{\min} = 0$$

Eq. (6-36): $\sigma_m = K_f \dfrac{\sigma_{\max} + \sigma_{\min}}{2} = (1.96)\dfrac{(43.7 + 0)}{2} = 42.8$ kpsi

$$\sigma_a = K_f \left|\frac{\sigma_{\max} - \sigma_{\min}}{2}\right| = (1.96)\left|\frac{(43.7 - 0)}{2}\right| = 42.8 \text{ kpsi}$$

Eq. (6-46): $\dfrac{1}{n_f} = \dfrac{\sigma_a}{S_e} + \dfrac{\sigma_m}{S_{ut}} = \dfrac{42.8}{42.0} + \dfrac{42.8}{120}$

$$n_f = 0.73 \qquad \textit{Ans.}$$

A factor of safety less than unity indicates a finite life.

Check for yielding. It is not necessary to include the stress concentration for static yielding of a ductile material.

$$n_y = \frac{S_y}{\sigma_{\max}} = \frac{66}{43.7} = 1.51 \qquad Ans.$$

6-25 Given: $F_{max} = 28\ kN, F_{min} = 0kN$. From Table A-20, for AISI 1040 CD, $S_{ut} = 590$ MPa, $S_y = 490$ MPa,

Check for yielding

$$\sigma_{\max} = \frac{F_{\max}}{A} = \frac{28\,000}{10(25-6)} = 147.4 \text{ N/mm}^2 = 147.4 \text{ MPa}$$

$$n_y = \frac{S_y}{\sigma_{\max}} = \frac{490}{147.4} = 3.32 \qquad Ans.$$

Determine the fatigue factor of safety based on infinite life

Eq. (6-8): $S_e^{'} = 0.5(590) = 295$ MPa

Eq. (6-19): $k_a = aS_{ut}^b = 4.51(590)^{-0.265} = 0.832$

Eq. (6-21): $k_b = 1$ (axial)

Eq. (6-26): $k_c = 0.85$

Eq. (6-18): $S_e = k_a k_b k_c S_e^{'} = (0.832)(1)(0.85)(295) = 208.6$ MPa

Fig. 6-20: $q = 0.83$

Fig. A-15-1: $d/w = 0.24,\ K_t = 2.44$

$$K_f = 1 + q(K_t - 1) = 1 + 0.83(2.44 - 1) = 2.20$$

$$\sigma_a = K_f \left|\frac{F_{max} - F_{min}}{2A}\right| = 2.2 \left|\frac{28000 - 0}{2(10)(25-6)}\right| = 162.1$$

$$\sigma_m = K_f \frac{F_{max} + F_{min}}{2A} = 162.1$$

$$\frac{1}{n_f} = \frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{162.1}{208.6} + \frac{162.1}{590} = 0.95$$

Since infinite life is not predicted, estimate the life from the *S*-*N* diagram.

$$\sigma_{rev} = \frac{\sigma_a}{1-(\sigma_m/\sigma_{ut})} = \frac{162.1}{1-(\frac{162.1}{590})} = 223.5$$

Fig. 6-18: $f = 0.87$

Eq. (6-14): $a = \dfrac{(f\,S_{ut})^2}{S_e} = \dfrac{\left[0.87(590)\right]^2}{208.6} = 1263$

Eq. (6-15): $b = -\dfrac{1}{3}\log\left(\dfrac{f\,S_{ut}}{S_e}\right) = -\dfrac{1}{3}\log\left(\dfrac{0.87(590)}{208.6}\right) = -0.1304$

Eq. (6-16): $N = \left(\dfrac{\sigma_{rev}}{a}\right)^{\frac{1}{b}} = \left(\dfrac{223.6}{117.8}\right)^{\frac{1}{-0.1304}} = 58(10^3)\ cycles$
